The γ-k_L Model for Prediction of Transitional Flow over a Flat Plate with Zero Pressure Gradient

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Abstract

Recently, two popular transition models have been proposed for *RANS-based* simulations: one is the correlation-based γ-Re_θ model [1-4] and the other is the physics-based k_L model [5-7]. The former is based on the intermittency concept to control the turbulent production/destruction terms in the existing turbulence model. The intermittency factor γ requires some mechanisms to predict the onset and length of transition which stem from empirical correlations. The latter is based on a second kind of kinetic energy, the laminar kinetic energy k_L, to deal with the transition process. The onset of transition is clearly defined through some parameters, similar to the vorticity Reynolds number, and the length of transition is modeled by transferring energy from laminar kinetic energy to turbulent kinetic energy. The transition process is hence modeled physically. In addition, different base turbulence models are used in these two transition models. The γ-Re_θ model uses the *SST-k-ω* turbulence model [8] without any modification while the k_L model uses a modified form of the standard k-ω model that contains many damping functions responsible for transitional effects. It is found that the intermittency concept is good for developing a new *RANS-based* transition model without modifying the turbulence model. The laminar kinetic energy concept, however, sounds more physical. Therefore, the present paper is aimed to propose a new transition model by using the intermittency concept via a physics-based approach. The present work uses the *SST-k-ω* [8] as a base turbulence model. The additional equations are the transport equations for the intermittency and laminar kinetic energy. The proposed model is evaluated by comparing its results with those of two transition models cited above and also the experimental data [9].

Keywords*:* SST k-ω; Transition; Intermittency; Laminar Kinetic Energy; RANS

1. Introduction

The *SST-k-ω* turbulence model of Menter [8] is widely accepted for predicting turbulent flow in many engineering applications. Recently, Menter's group has modified and enables it to predict transitional flow by using an intermittency concept. Menter's group proposed the $\gamma - \mathrm{Re}_\theta$ transition model that consists of γ- and $\tilde{\mathrm{Re}}_\theta$-transport equations. The γ is the intermittency function in which the information is contributed from $\tilde{\mathrm{Re}}_\theta$. Their concept is very interesting. There is a little change in the base turbulence model used [8]. Only the production and destruction terms in the turbulent kinetic energy equation are modified by using the intermittency function as a weighting factor. However, this model is based on empirical correlations and there are two important parameters that were not published at the first time. As a result, now there are many correlation parameters proposed in the literature. It is still a mystery if the two parameters just disseminated by Menter's group [4] can be compatible with different in-house CFD codes. Recently, nevertheless, Walters and co-workers, i.e., Walter and Leylek [5], Walters and Leylek [6], and Walters and Cokljat [7], proposed a new transition model using the concept of laminar kinetic energy. It is based on physics of transitional flow. Moreover, all functions and terms are given and clearly defined. Therefore, this model can work immediately in any in-house CFD codes.

The objective of this work is to incorporate the laminar kinetic energy into the base *SST-k-ω* turbulence model in terms of the intermittency factor. The intermittency factor will be obtained from the newly proposed intermittency transport equation and the original version of the laminar kinetic energy transport equation in Walters and Cokljat [7], hereafter called the *Walters model*, is employed. All functions and constants also stem from the Walters model. The concept of transition-turbulence modeling connection is similar to what Menter's group did for the $\gamma - \mathrm{Re}_\theta$ model, hereafter called the *Menter model*. Thus the base turbulence model will be modified as little as possible.

2. Transition Models

There are two transition models involved in this paper. One is the transition model of Menter's group [1-4] that is based on empirical correlations with an intermittency factor concept and the other is the Walters model [7] that is based on the physics of transitional flow with a laminar kinetic energy concept. Due to the limitation of page number, both models are briefly described in the following sub-sections.

2.1 Menter and Langtry Transition Model: Menter Model

This model uses an intermittency factor to control the growth rate of turbulent kinetic energy through the production and destruction terms in the turbulent kinetic energy equation in the existing turbulence model. The turbulence model used is the *SST-k-ω* model [8]. The turbulent kinetic energy equation incorporated with the intermittency factor can be written as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j k) - \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \mu_t\right)\frac{\partial k}{\partial x_j}\right] = \tilde{P}_k - \tilde{D}_k \qquad (1)$$

where

$$\tilde{P}_k = \gamma P_k \tag{2}$$

$$\tilde{D}_k = \min\left(\max(\gamma, 0.1), 1.0\right) D_k \tag{3}$$

The intermittency factor γ is obtained from the following modeled transport equation:

$$\frac{\partial}{\partial x_j}(\rho u_j \gamma) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_f}\right)\frac{\partial \gamma}{\partial x_j}\right] = P_\gamma - E_\gamma \tag{4}$$

where

$$P_\gamma = F_{length} c_{a1} \rho S [\gamma F_{onset}]^{0.5} (1 - c_{e1}\gamma) \tag{5}$$

$$E_\gamma = c_{a2} \rho \Omega F_{turb} (c_{e2}\gamma - 1) \tag{6}$$

The initiation of transition is triggered by the parameter F_{onset}, while the length of transition is controlled through the parameter F_{legnth}. Both parameters are the function of a local transition onset momentum-thickness Reynolds number, $\tilde{\mathrm{Re}}_{\theta t}$, that is obtained from the following transport equation:

$$\frac{\partial}{\partial x_j}(\rho u_j \tilde{\mathrm{Re}}_{\theta t}) - \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu + \mu_t)\frac{\partial \tilde{\mathrm{Re}}_{\theta_t}}{\partial x_j}\right] = P_{\theta t} \tag{7}$$

where

$$P_{\theta t} = c_{\theta t}\frac{(\rho U)^2}{500\mu}(\mathrm{Re}_{\theta t} - \tilde{\mathrm{Re}}_{\theta t})(1 - F_{\theta t}) \tag{8}$$

However, the (actual) transition onset momentum-thickness Reynolds number, $\mathrm{Re}_{\theta t}$, is obtained from the empirical correlation. Further detail of this model can be found in [4].

2.2. Walters and Cokljat Transition Model: Walters Model

This model can be called the three-equation transition model and can be summarized as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j k_T) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\alpha_T}{\sigma_k}\right)\frac{\partial k_T}{\partial x_j}\right] = P_{k_T} + R_{BP} + R_{NAT} - \rho\omega k_T - D_T \tag{9}$$

$$\frac{\partial}{\partial x_j}(\rho u_j k_L) - \frac{\partial}{\partial x_j}\left[\mu\frac{\partial k_L}{\partial x_j}\right] = P_{k_L} - R_{BP} - R_{NAT} - D_L \tag{10}$$

$$\begin{aligned}\frac{\partial}{\partial x_j}(\rho u_j \omega) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\alpha_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] &= \rho C_{\omega 1}\frac{\omega}{k_T}P_{k_T} + \rho\left(\frac{C_{\omega R}}{f_W} - 1\right)(R_{BP} + R_{NAT}) \\ &\quad - \rho C_{\omega 2}\omega^2 + \rho C_{\omega 3} f_\omega \alpha_T f_W^2 \frac{\sqrt{k_T}}{d^3}\end{aligned} \tag{11}$$

where k_T is the turbulent kinetic energy, k_T is the laminar kinetic energy, and ω is the specific dissipation rate of turbulent kinetic energy. In this model, the turbulence or eddy is divided into two parts: large- and small-scale ones by the effective turbulence length scale,

$$\lambda_{eff} = \min(C_\lambda d, \lambda_T) \tag{12}$$

where d is the normal nearest wall distance and λ_T is the turbulence length scale. Then the viscous damping function, f_W, is used to divide the turbulent kinetic energy into the small scale $k_{T,s}$ and large scale $k_{T,l}$ as follows:

$$f_W = \left(\frac{\lambda_{eff}}{\lambda_T}\right) \tag{13}$$

$$k_{T,s} = f_{SS} f_W k_T \tag{14}$$

$$k_{T,l} = k_T - k_{T,s} \tag{15}$$

where f_{SS} is the shear-sheltering effect proposed in [7] to account for transitional initiation and is defined as

$$f_{SS} = \exp\left[-\left(C_{SS}\frac{\nu\Omega}{k_T}\right)^2\right] \quad (16)$$

The turbulent kinetic energy production term is defined as

$$P_{k_T} = \nu_{T,s}S^2 \quad (17)$$

The laminar kinetic energy production term can also be defined in a similar form as

$$P_{k_L} = \nu_{T,l}S^2 \quad (18)$$

where the small-scale and large-scale eddy viscosities in Eqs. (17) and (18) are modeled respectively as follows:

$$\nu_{T,s} = f_\nu f_{INT} C_\mu \sqrt{k_{T,s}} \cdot \lambda_{eff} \quad (19)$$

$$\nu_{T,l} = \min\left\{ f_{\tau,l} C_{l1}\left(\frac{\Omega\lambda_{eff}^2}{\nu}\right)\sqrt{k_{T,l}} \cdot \lambda_{eff} + \beta_{TS} C_{l2} \mathrm{Re}_\Omega d^2\Omega, \frac{0.5\cdot(k_L + k_{T,l})}{S}\right\} \quad (20)$$

The terms R_{BP} and R_{NAT} in Eqs. (9) - (11) are called the redistribution terms which are responsible for energy transfer from laminar kinetic energy to turbulent kinetic energy as can be seen by different signs in Eqs. (9) and (10). These terms are modeled in the following forms:

$$R_{BP} = C_R \beta_{BP} k_L \omega / f_W \quad (21)$$

$$R_{NAT} = C_{R,NAT} \beta_{NAT} k_L \Omega \quad (22)$$

The criteria where these terms start transferring energy are given by the following functions:

$$\beta_{BP} = 1 - \exp\left(-\frac{\phi_{BP}}{A_{BP}}\right) \quad (23)$$

$$\phi_{BP} = \max\left[\left(\frac{k_T}{\nu\Omega} - C_{BP,crit}\right), 0\right] \quad (24)$$

$$\beta_{NAT} = 1 - \exp\left(-\frac{\phi_{NAT}}{A_{NAT}}\right) \quad (25)$$

$$\phi_{NAT} = \max\left[\left(\frac{d^2\Omega}{\nu} - \frac{C_{NAT,crit}}{f_{NAT,crit}}\right), 0\right] \quad (26)$$

$$f_{NAT,crit} = 1 - \exp\left(-C_{NC}\frac{\sqrt{k_L}\cdot d}{\nu}\right) \quad (27)$$

Coefficients, constants and functions that are not given here are referred to the original paper [7].

3. Present Model Concept

The turbulent production term of the Walters model, Eq. (9), can be arranged as the following form

$$P_{k_T} = f_{WC}\left(\nu_T S^2\right) \quad (28)$$

where

$$f_{WC} = f_\nu f_{INT} f_{SS}^{1/2} f_W^{3/2} \quad (29)$$

It is found that the term in parentheses is the standard form of the turbulent production term, while f_{WC} is a collection of damping functions that come from the Walters model. Therefore, the term f_{WC} can be viewed as an intermittency factor. Consequently, the turbulent kinetic energy equation, Eq. (1), can be rearranged to explicitly include the intermittency factor f_{WC} as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j k) - \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \mu_t\right)\frac{\partial k}{\partial x_j}\right] = f_{WC} P_k - C_{WC} f_{WC} D_k \quad (30)$$

The factor f_{WC} is just simply multiplied to the destruction term because f_{WC} affects to the dissipation rate of turbulent kinetic energy equation through the turbulent production terms P_k. Therefore, it can be included in Eq. (30), with C_{WC} as a constant of proportionality, without modifying the ω-equation. To account for the redistribution terms, R_{BP} and R_{NAT}, the factor in Eq. (29) is modeled as follows:

$$\gamma = f_v f_{INT} f_{SS}^{1/2} f_W^{3/2} + \frac{(R_{BP} + R_{NAT})}{\nu_T S^2} \tag{31}$$

where γ is used in Eq. (31) instead of f_{WC}. Eq. (31) can be used as the intermittency factor expressed in an algebraic form. In Ref. [10], this algebraic form of intermittency factor was applied to the *SST* turbulence model with the laminar kinetic energy equation, Eq. (10). Some achievements were present there. Hereafter, it will be called the *algebraic model*.

The construction of an intermittency transport equation actually starts from Eq. (28). If the turbulent kinetic energy equation incorporated with the Walters model damping functions [7] and the intermittency factor can be written in the following form:

$$\frac{\partial}{\partial x_j}(\rho u_j k) - \frac{\partial}{\partial x_j}\left[(\mu + \sigma_k \mu_t)\frac{\partial k}{\partial x_j}\right] = \gamma\left(f_{WC} P_k\right) - \gamma\left(\rho \omega k\right) \tag{32}$$

Then the intermittency transport equation can mimic Eq. (32) and be modeled as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j \gamma) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_f}\right)\frac{\partial \gamma}{\partial x_j}\right] = \gamma\left(f_{WC} P_k\right)\frac{\gamma}{k} - \gamma\left(\rho \omega k_L\right)\frac{(1-\gamma)}{k} \tag{33}$$

The last term on the right hand side is multiplied by factor the *(1-γ)* because this term should be activated only in a non-turbulent region and has no effect in a fully turbulent one. For the same reason, k_L is used instead of *k* in the destruction term. Finally, the redistribution terms are included in Eq. (33) in a similar way as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j \gamma) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_f}\right)\frac{\partial \gamma}{\partial x_j}\right] = \gamma\left(f_{WC} P_k\right)\frac{\gamma}{k} - \gamma\left(\rho \omega k_L\right)\frac{(1-\gamma)}{k} + \frac{\gamma}{k}\left(R_{BP} + R_{NAT}\right) \tag{34}$$

This is the final form of the present intermittency transport equation and hereafter it is called the *transport model*. However, it is necessary to restrict its value not to exceed unity during the calculation.

4. Computational Setup

All the results presented here were generated by the in-house CFD code that has been validated for years. The code uses the finite volume method based on a structured Cartesian mesh. The domain is sub-divided into many finite control volumes as shown in Fig. 1. High mesh resolution is applied around the leading edge, at x=0, and also close to the wall. The domain size is 1.75x0.2 m^2 with 0.05 m extended upstream of the leading edge. The mesh used in all calculations contains 180x120 control volumes. The *QUICK* convection scheme is used here in the momentum equation, while a typical 1st order upwind scheme is used in other transport equations. The *SIMPLE* algorithm is used to solve the system of algebraic equations for pressure-velocity coupling. The boundary conditions for *k* and ω are the same as those in the *SST-k-ω* model. For k_L, the no-slip condition is applied, that is, $k_L=0$ at the wall. The zero normal gradient is applied for γ at wall. At inlet, k_{in}=1.5$(Tu_\infty \cdot U_\infty)^2$, $\mu_t=R_\mu \cdot \mu$, $\omega_{in}=\rho k_{in}/\mu_{t,in}$, $k_{L,in}=0$, and $\gamma=1$ are used. At free-stream and outlet, the zero normal gradient is applied for all variables. The computations are divided into three cases, each of which has different free-stream velocity (U_0) and free-stream turbulent intensity (Tu) as shown in Table 1. The highest Tu is classified as a bypass transition, while the lowest one is classified as natural transition.

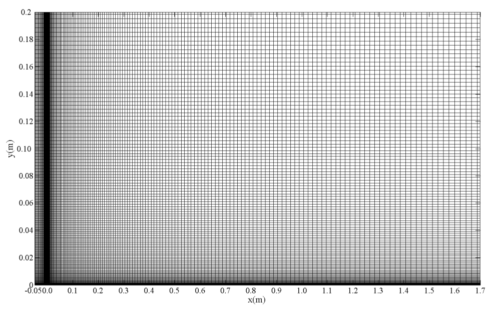

Fig. 1 Mesh distribution (not to scale)

Table. 1 Inlet conditions used in each case

CASE	U_0 (m/s)	Tu (%)	μ_t/μ
T3B	9.4	6.5	100.0
T3A	5.4	3.5	12.0
T3AM	19.8	0.874	8.73

5. Results and Discussion

The decays of turbulent kinetic energy at three different free-stream turbulence intensities are shown in Fig. 2. They are validated with the experimental data. The present results agree very well with the reference data.

In order to ensure that the laminar kinetic energy k_L plays its role in the present model, the distribution of the ratio between the turbulent kinetic energy k and the total kinetic energy, $k + k_L$, along the wall are monitored in Fig. 3. Such distribution can roughly indicate the point where the transition starts, as shown by arrows, in the present computation. As shown in Fig. 3, the lower the ratio, the larger the magnitude of the laminar kinetic energy, therefore the k_L has played its role in the proposed model.

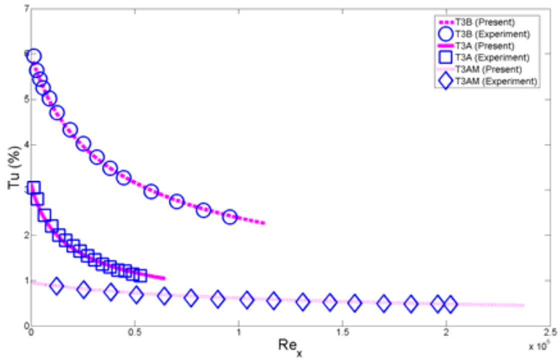

Fig. 2 Free-stream turbulence intensities of T3B, T3A and T3AM cases.

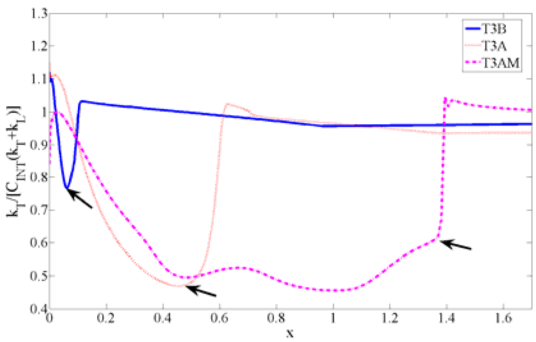

Fig. 3 Distribution of $k/[C_{INT}(k+k_L)]$ along the wall

Figs. 4 – 6 show the proposed transport model results of skin friction coefficient in case of zero pressure gradient compared to those of the Menter model with proposed parameters of Suluksna et al [11] and of the Walters model. In addition, the results of the algebraic model are also included for comparison. The T3B case is shown in Fig. 4 for the free-stream turbulence intensity of Tu = 6.5%. The prediction of proposed transport model is slightly more accurate than the Menter and algebraic models, but not as good as the Walters model. The onset point occurs quite early and the minimum value of C_f cannot be captured. However, its tendency is closer to the Walter model than the Menter model and the algebraic model, which is the desired target. The T3A case, *Tu = 3.5%*, is shown in Fig. 5. The proposed transport model predicts more accurate than the algebraic and Walters model, but it captures the transition length slightly less accurate than the Menter model. However, the proposed transport model can detect the onset point quite well. In Fig. 6, the T3AM case is shown at *Tu = 0.874%*. The proposed transport model fails to predict the length of transition. As shown, the C_f increases steeply from the laminar line up to turbulent one. However, the proposed transport result can predict more accurate than the algebraic and Walters models. This seems to be the drawback of the Walters model that is not good at prediction of low free-stream turbulence intensity case or natural transition and the proposed transport model uses almost all the formation from the Walters model. Therefore, it is not wonder why the proposed transport model predict not as accurately as the former cases. From all the presented results, in summary, the present γ-k_L transport model can work well at moderate free stream turbulent intensity. For the high free-stream turbulent intensity, however, the proposed transport model is affected from the use of the *SST-k-ω* turbulence model in which the Menter transition model also cannot predict well in this case. In contrast, for the low *Tu* case, the proposed transport model is likely as accurately as the Walters model.

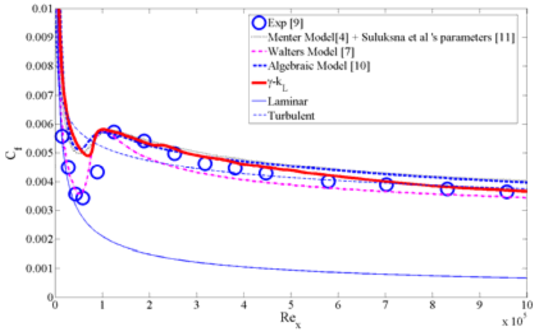

Fig. 4 Skin friction coefficient for T3B, *Tu = 6.5 %*

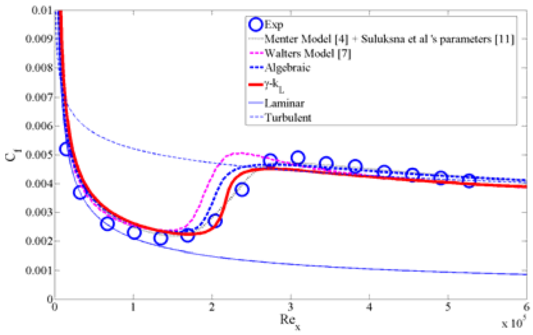

Fig. 5 Skin friction coefficient for T3A case, *Tu = 3.5 %*

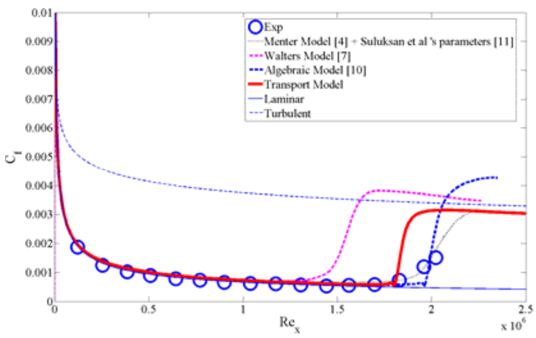

Fig. 6 Skin friction coefficient for T3AM case, *Tu = 0.874%*

6. Conclusion

New modeling of the intermittency transport equation has been presented and incorporated into the *SST-k-ω* turbulence model. The predicted results in case of the flat plate zero pressure gradient flow is quite well and reasonable. However, it shows some feasibility that the proposed model will be capable in the near future. On the other hand, the new γ-k_L transition model can be summarized as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j \gamma) - \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_f}\right)\frac{\partial \gamma}{\partial x_j}\right] = \gamma\left(f_{WC} P_k\right)\frac{\gamma}{k} - \gamma\left(\rho \omega k_L\right)\frac{(1-\gamma)}{k} + \frac{\gamma}{k}\left(R_{BP} + R_{NAT}\right)$$

Then it is incorporated into the turbulent kinetic energy as

$$\frac{\partial}{\partial x_j}(\rho u_j k) - \frac{\partial}{\partial x_j}\left[(\mu + \sigma_k \mu_t)\frac{\partial k}{\partial x_j}\right] = \gamma P_k - \max(\gamma, 0.1) D_k$$

The last term on the right hand side uses the same criteria as the Menter model [4]. The laminar kinetic energy equation used is in the following form:

$$\frac{\partial}{\partial x_j}(\rho u_j k_L) - \frac{\partial}{\partial x_j}\left[\mu \frac{\partial k_T}{\partial x_j}\right] = P_{k_L} - R_{BP} - R_{NAT}$$

The *SST-k-ω* model is not modified. The terms and functions that belong to the Walters model [7] are directly adopted.

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8. References

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