

## Design Optimization of Plate-Fin Heat Sinks Using Hybridization of MPSO and RSM

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### **Abstract**

The work in this paper is aimed at demonstrating multiobjective optimization of plate-fin heat sinks and the superiority of combining a response surface method and multiobjective evolutionary optimizer over solely using the evolutionary optimizer. The design problem is assigned to minimize a heat sink junction temperature and fan pumping power. Design variables determine heat sink geometry. Design constraints are given in such a way that the maximum and minimum fin heights are properly limited. The function evaluation is carried out by using the finite volume analysis software. Two multiobjective evolutionary optimization strategies, multiobjective particle swarm optimizers with and without the use of a response surface technique, are implemented to explore the Pareto optimal front. The optimum results obtained from both design approaches are compared and discussed. It is illustrated that the multiobjective evolutionary technique is a powerful tool for the multiobjective design of the electronic air-cooled heat sinks. With the same design conditions and number of function evaluation, the multiobjective particle swarm optimizer with the use of the response surface technique totally outperforms the other.

**Key words:** Multiobjective particle swarm optimizer, Response surface method, Plate-fin heat sink, Geometrical design, Finite volume method.

### **1. Introduction**

Due to the growth trend in integrated circuit (IC) technology recently, electronic cooling systems and components with higher performance are needed. Higher power density and heat dissipation as well as a decreased size of those components are expected. The future electronic products need to be long-lasting and

capable of handling the more severe environment [1]. As a result, thermal management and design in packaging industry becomes increasingly important. An aluminum air-cooled heat sink is one of the most commonly used components for cooling electronic packages. Using such a cooling system is advantageous in that it has a simple maintenance process, more reliability, lower manufacturing cost and no environmental



concerns compared to other cooling methods. It has been studied by many researchers that the heat-sink with good geometrical design provides better cooling performance. This means that the optimization process could be an effective design tool for the heat sink. A lot of research work has been conducted in the field of heat sink design/optimization e.g. in [1-7]. It is obvious from the literature survey that most of the optimization studies are limited to single objective function whereas, in reality, there are multiple objectives to be decided. If the multiobjective design of heat sinks can be achieved, it could make a significant impact on the electronic packaging technology. The successful use of multiobjective evolutionary optimizers for practical problems has been reported for many years. Using such optimizers is advantageous as they are simple to implement, need no function derivatives, and can deal with almost all kinds of design functions and variables. Moreover, the multiobjective evolutionary optimization process can hardly stall. The most outstanding ability of the multiobjective evolutionary optimizers is that they can explore a Pareto optimum set within one simulation run. They however have some undesirable drawbacks, which are a lack of consistency and slow convergence rate. The optimizers are said to be unsuitable for a design problem with highly expensive function evaluation. As a result, the hybridisation of a response surface method and the multiobjective evolutionary optimizers is invented and this approach is found to be greatly powerful and effective [8]. In this study, the geometrical design of plate-fin heat sinks is carried out using the multiobjective particle swarm optimizer with and without the

hybridisation of a response surface method. The design problem is assigned to find heat sink geometries such that minimising a heat sink junction temperature and fan pumping power. Design constraints are given in such a way that the maximum and minimum fin heights are properly limited. The function evaluation is achieved by using the CFD software. Two multiobjective strategies are implemented to explore the Pareto optimal front. The optimum results obtained from both design approaches are compared. It is illustrated that, with the same design conditions and number of function evaluations, the evolutionary multiobjective optimizer with the use of the response surface technique is far superior. The applied multiobjective evolutionary technique is a powerful tool for the design of electronic air-cooled heat sinks.

## **2. Multiobjective Particle Swarm Optimizer**

The multiobjective evolutionary optimizer used to find the Pareto optimum solutions in this research work is multiobjective particle swarm optimization (MPSO). The method of particle swarm optimization has been recently used as a population-based or evolutionary optimizer for both single- and multiple- objective design cases. The method is said to be simple but found to be effective compared to some other evolutionary algorithms. It can be thought of as mimicking the movement of a flock of birds which aim to find food [9]. For multiobjective optimization, the search procedure starts with an initial set of design solutions along with their corresponding initial particle velocities. Having evaluated the objective function values of the individuals in the

initial population, the non-dominated solutions of the initial population are taken to an initial external Pareto archive. The new set of solutions is found by using the following updating strategy:

$$\mathbf{x}_i(k) = \mathbf{x}_i(k-1) + \mathbf{v}_i(k) \quad (1)$$

and

$$\mathbf{v}_i(k) = W\mathbf{v}_i(k-1) + C_1 r_1 (\mathbf{p}_i^{best} - \mathbf{x}_i(k)) + C_2 r_2 (\mathbf{g}_i^{best} - \mathbf{x}_i(k)) \quad (2)$$

Where  $\mathbf{x}_i(k)$  is the  $i^{\text{th}}$  individual at the  $k^{\text{th}}$  iteration  
 $\mathbf{v}_i(k)$  is the velocity vector of  $\mathbf{x}_i(k)$  at the  $k^{\text{th}}$  iteration

$W$  is an initial weight used to control the impact of the previous velocities

$r_1, r_2 \in [0,1]$  are a uniform random number

$C_1$  is called a cognitive learning factor

$C_2$  is called a social learning factor

$\mathbf{p}_i^{best}$  is the personal best of the individual  $\mathbf{x}_i$

$\mathbf{g}_i^{best}$  is the global best solution.

In this paper,  $\mathbf{g}_i^{best}$  are randomly selected from the external archive. The non-dominated solutions of the union set of the new population and the non-dominated solutions in the previous Pareto archive are sorted and saved to the new external Pareto archive. In cases where the number of non-dominated solutions exceeds the archive limit, the adaptive grid algorithm (see [10]) is operated to properly remove some non-dominated solutions from the archive. Fig. 1 demonstrates how the adaptive grid technique for the bi-objective case works. Having generated a grid covering all of the non-dominated solutions, one of the members in the most crowded region is removed from the archive. The crowded regions are updated and the member in the most

crowded region is removed iteratively until the number of non-dominated solutions is equal to the size of the archive. The search procedure is repeated until fulfilling a termination criterion.

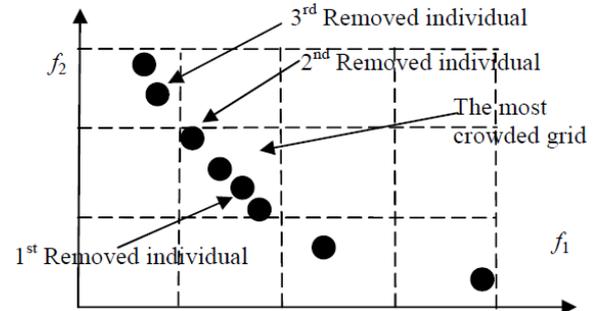


Fig. 1. Adaptive grid algorithm

### 3. Hybridization of MPSO with a Response Surface Method

A response surface method is a proven numerical strategy for use in optimization process. The successful results in using RSM for both single and multiple objective optimizations have been reported worldwide [8]. The basic concept is to exploit numerical curve fitting or interpolation to approximate non-linear design objectives or constraints. The optimization is carried out using the approximate model rather than using the real function evaluation. This means that the problem of time-consuming function evaluation can be alleviated. However, it also has some disadvantages e.g. inaccurate function approximation may lead evolutionary search to the improper region of design space or even away from the real optima. Coupling RSM with MPSO search can be achieved by using MPSO as the main procedure. As a population is created, some design points in the population are taken to build an approximate design problem. Then, another MPSO sub-optimizer is used to explore the Pareto optimal set using the approximate function model. The Pareto optimal solutions obtained from the

use of the sub-optimizer and the RSM model are taken back to the main optimization procedure. The real function values of the selected non-dominated solutions from optimizing the approximate model are evaluated. The non-dominated solutions from the previous external archive, the current population and the design points taken from optimizing the RSM model are put together and sorted to have the new set of non-dominated solutions. The external Pareto archive is then updated by using the new set of non-dominated solutions. This process is repeated until the termination criterion is fulfilled. The numerical procedure of a coupled MPSO and RSM for an unconstrained case is illustrated in Fig. 2. In the optimization process using the RSM model, at the  $t^{\text{th}}$  iteration, design solutions are represented by  $\mathbf{y}_i(t)$ . Particles' velocities are denoted by  $\mathbf{u}_i(t)$ . An external Pareto archive is denoted by  $\mathbf{B}^t$  whereas  $\mathbf{g}(\mathbf{y}_i(t))$  are function values computed using RSM.

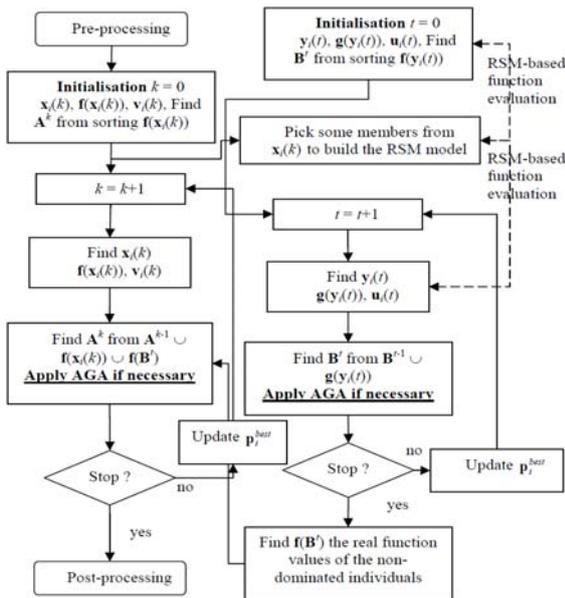


Fig. 2. Flowchart for coupled MPSO and RSM

#### 4. Multiobjective Design Problem

The thermal performance and required pumping power of an air-cooled heat sink are dependent on a number of parameters including fin thickness, fin height, number of fins, fin to fin spacing, approach air velocity and base-plate dimension. Fig. 3 displays the shape of a typical plate-fin heat sink and some defined parameters. The variation of fin heights can also be assigned as design variables. In this work, the multiobjective design problem of the plate-fin heat sink is posed as follows

$$\min_{\mathbf{X}} \mathbf{f}(\mathbf{X}) = \{f_1(\mathbf{X}) \quad f_2(\mathbf{X})\} \quad (3)$$

Subjected to

$$10 - H_{\min}/b \leq 0$$

$$H_{\max}/b - 25 \leq 0$$

where  $f_1$  is a junction temperature.

$f_2$  is a fan pumping power.

$H_{\max}$  is the maximum fin height.

$H_{\min}$  is the minimum fin height.

The first objective function is a temperature value at the junction of the heat sink and a CPU chip, which can be written as

$$f_1(\mathbf{X}) = T_{jc} = T_a + QR_{HS} \quad (4)$$

where  $T_{jc}$  is a junction temperature

$T_a$  is an ambient air temperature (298 K)

$Q$  is heat flux (80 watts)

$R_{HS}$  is heat sink thermal resistance.

The minimisation of this objective value indicates the thermal performance of the heat sink. The second objective function affects the fin cost, which is rather inevitable in any engineering system design. The fan pumping power can be expressed as:

$$f_2(\mathbf{X}) = P_F = \frac{\dot{m}_a \Delta P}{\rho_a} \quad (5)$$

where  $P_f$  is a fan pumping power

$\dot{m}_a = WLV_f$  is an air flow rate

$\Delta P$  is a pressure drop across the heat sink

$\rho_a$  is air density

$V_f$  is an inlet air velocity.

Fig. 4 illustrates some of the design parameters that determine the heat sink cross-sectional area. The vector  $\mathbf{X}$  of seven design variables can be detailed as

$X_1 = d$ : fin thickness having the bound as [0.5,3.0] mm.

$X_2 = n$ : fin number having the bound as [5,30]

$X_3 = t_b$ : plate base thickness having the bound as [1.0,5.0] mm

$X_4 = H_1$ : fin height in fig.4 having the bound as [25,140] mm

$X_5 = H_2$ : fin height in fig.4 having the bound as [25,140] mm

$X_6 = H_3$ : fin height in fig.4 having the bound as [25,140] mm

$X_7 = V_f$ : inlet air velocity having the bound as [0.5,6] m/s

The values of  $H_1$ ,  $H_2$  and  $H_3$  control the fin height variation. The parameters  $W$  and  $L$  are set to be 60 mm and 80 mm respectively.

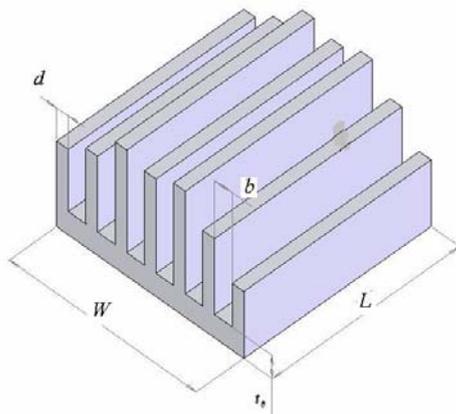


Fig. 3 Plate-fin heat sink

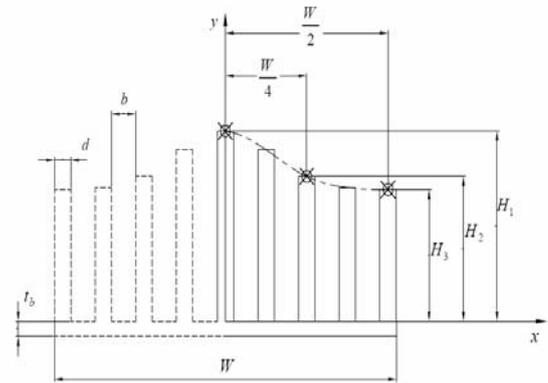


Fig. 4 Design parameters

The thermo-fluid analysis of the forced convection plate-fin heat sink is achieved by using the finite volume method. The finite volume model of the heat sink is (both fluid and solid domains) is given in Fig. 5, the assumptions for this analysis are as follows

- Fluid flow being laminar and steady
- Constant material thermo-physical of both air and solid
- Uniform approach air velocity
- Uniform heat flux entire base plate bottom surface.

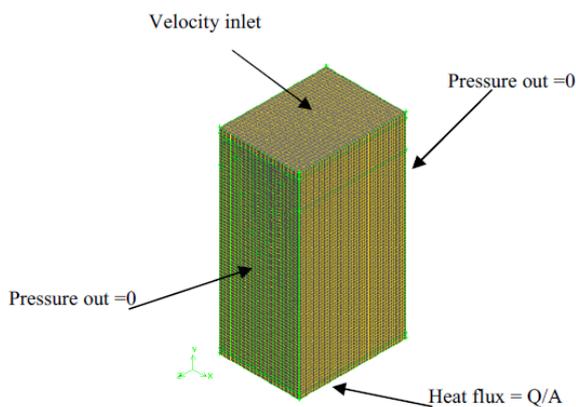
The heat sink body is made of Aluminum. The physical properties of the solid and fluid are given in table 1.

Table 1. Aluminium and air properties

Aluminum properties	Air Properties
Density = 2719 kg/m <sup>3</sup>	Density = 1.177 kg/m <sup>3</sup>
Specific heat = 871 J/kg-K	Specific heat = 1006 J/kg-K
Thermal conductivity = 202 W/m-K	Thermal conductivity = 0.0267 W/m-K
	Viscosity = 1.8832 × 10 <sup>-5</sup> kg/m-s

The multiobjective particle swarm optimizer without the use of RSM (termed MPSO without RSM) has the population sized 25 and uses 50 generations for exploring the Pareto front

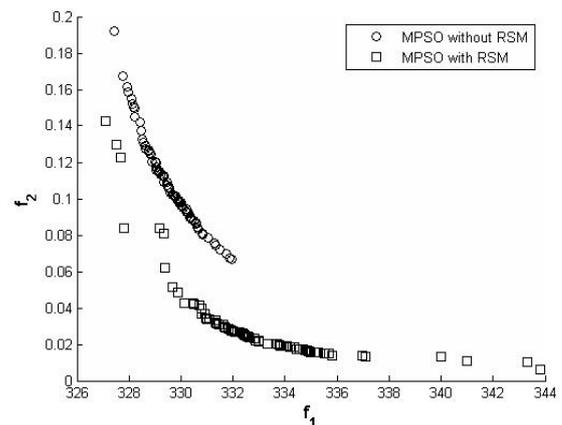
of the problem (3). For the MPSO with RSM method, the population size is set to be 25. On each generation, 25 individuals obtained from using the MPSO sub-optimizer with RSM function approximation are taken to the main optimization procedure while other 25 individuals are created from the main searching process of the multiobjective particle swarm optimization. In order to have the same number of finite volume analyses, MPSO with RSM needs 25 generations. The response surface method is based on the concept of multiquadric approximation [8]. The size of an external Pareto archive is set to be 80 for both optimization strategies. The methods start searching with the same initial solutions. The design constraints can be dealt with by using the non-dominated sorting concept presented in [11]. According to the population size and the number of generations, the total number of times that the finite volume analysis takes place is equal to  $25 \times 50$  for both optimization methods. It is obvious that the MPSO with RSM needs more computational time for the RSM sub-optimization process but it is insignificant compared to the long period of time needed for the finite volume analysis.



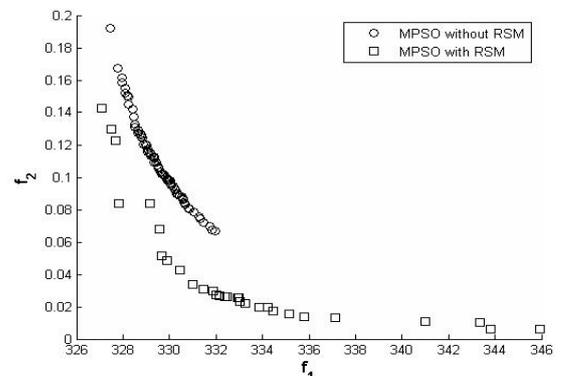
**Fig.5** Finite volume model of the heat sink

## 5. Design Results

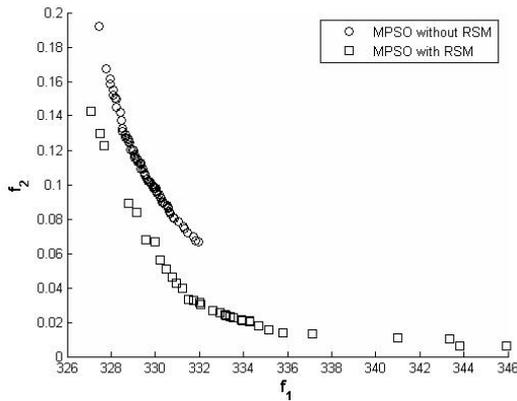
Fig. 6 shows the approximate Pareto fronts explored by the MPSO methods with and without RSM. It is clear that MPSO with RSM totally outperforms the other. Fig. 7 displays the comparison of the front obtained from the MPSO without RSM at the 50<sup>th</sup> iteration and the front obtained at the 10<sup>th</sup> iteration of the MPSO with RSM method. It can be seen that the later front is still far better. The more critical comparison is illustrated in Fig. 8, which is between the front of MPSO without RSM after  $25 \times 50$  finite volume analyses and the front obtained from MPSO with RSM after  $50 \times 5$  analyses. The latter is still far superior to the former.



**Fig.6** MPSO without RSM at 50<sup>th</sup> iteration VS MPSO with RSM at 25<sup>th</sup> iteration

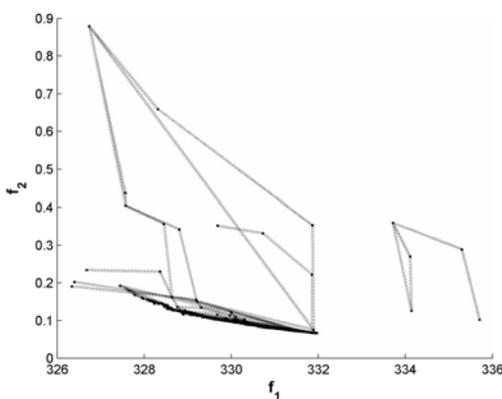


**Fig.7** MPSO without RSM at 50<sup>th</sup> iteration VS MPSO with RSM at 10<sup>th</sup> iteration

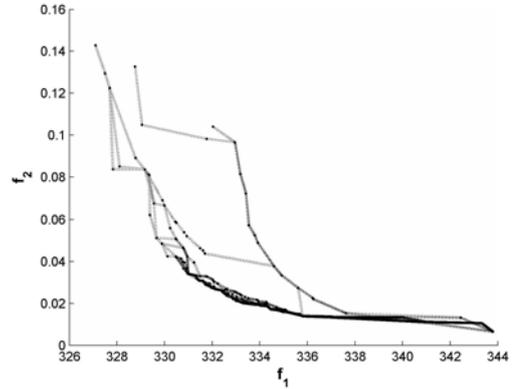


**Fig.8** MPSO without RSM at 50<sup>th</sup> iteration VS MPSO with RSM at 5<sup>th</sup> iteration

The progress in exploring the Pareto front of the design problem (3) by the MPSO without RSM method is shown in Fig. 9. It is shown that the set of non-dominated solutions was slowly improved until the final Pareto archive is obtained. The search history of the MPSO with RSM method is shown in Fig. 10. The non-dominated solutions approached the Pareto front rapidly and almost reached the Pareto front after the 5 generations. This shows the high convergence rate of the MPSO with RSM method.

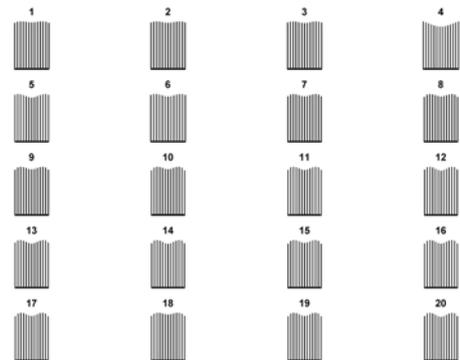


**Fig. 9** Search history of MPSO without RSM

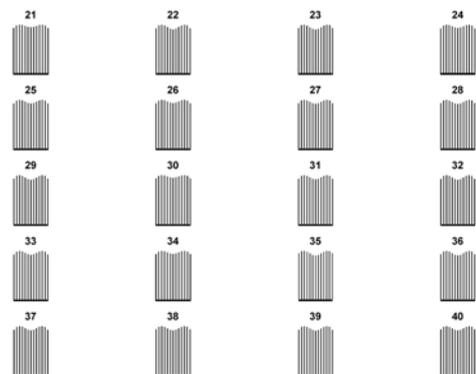


**Fig. 10** Search history of MPSO with RSM

The heat sink cross-sectional areas corresponding to the 40 sample Pareto points are illustrated in Figs. 11 – 12. Most of the fins have pretty similar fin height variation but different inlet velocity, number of fin, fin thickness, fin base thickness and consequently fin-to-fin space.



**Fig. 11** Fin cross-sectional areas of the points 1-20



**Fig. 12** Fin cross-sectional areas of the points 21-40

## 6. Conclusions and Discussion

The use of multiobjective particle swarm optimization with and without the combination of the response surface method for the multiobjective design of plate-fin heat sink geometry is demonstrated. From the obtained Pareto optimum results, the linked MPSO and RSM method is far superior to the MPSO method without RSM. The MPSO with RSM is a powerful design tool for multiobjective geometrical design of plate-fin heat sinks. With the use of such a method all aspects of design variables and functions can be dealt with and the optimization process can hardly stall.

## 7. Acknowledgements

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